

Quantum Physics-Informed Neural Network: A Surrogate Model for High-Order Δ - Σ Modulators

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Abstract—The Δ - Σ analog-to-digital converter (ADC), with the modulator as its core component, poses significant challenges due to complexity of high-order topologies and the associated computational cost in simulation and performance evaluation. While physics-informed neural networks (PINNs) have been introduced as a potential function approximator for a wide variety of systems, their effectiveness might be questioned when handling the highly nonlinear and high-dimensional dynamics of Δ - Σ modulators. Quantum neural networks (QNNs) offer enhanced capability for modeling complex systems by exploiting quantum superposition and entanglement, and therefore are promising candidates for PINNs. In this work, we propose a quantum physics-informed neural network (QPINN) as a surrogate model for predicting the signal-to-noise ratio (SNR) of high-order Δ - Σ modulators. The proposed framework involves a pre-processing stage of feature extraction to match the quantum circuit of 5 qubits. Simulation results demonstrate that the proposed QPINN achieves strong SNR prediction accuracy ($R^2=0.845$), making it suitable for rapid evaluation of high-order Δ - Σ modulator designs.

Index Terms— Δ - Σ modulator, surrogate model, physics-informed neural network, feature extraction, quantum neural network

I. INTRODUCTION

Δ - Σ analog-to-digital converters (ADCs) are widely used in high-resolution applications because they enable superior noise-shaping and signal integrity through oversampling [1], [2]. They are extensively used in audio processing, biomedical instrumentation, and wireless communication systems where high dynamic range and precision are required, especially in the wake of technology advancement, circuit design and layout implementation automation [3]. However, as performance requirements continue to increase, designers often rely on higher-order Δ - Σ modulators to further suppress in-band noise and improve the signal-to-noise ratio (SNR). This inevitably introduces significantly more complex topologies, leading to an exponential growth in the design space and stronger sensitivity to parameter variations. Consequently, circuit optimization, simulation and verification become computationally more expensive and time-consuming than the primitive blocks [4], especially when repetitive evaluations are required during design optimization. To address this problem, surrogate modeling techniques, including physics-informed neural networks (PINNs), have been explored as efficient alternatives to traditional simulation tools [5], [6]. By embedding circuits' topologies into neural networks, PINNs aim to precisely approximate system behavior. Nevertheless,

the intrinsic nonlinearity and high dimensionality of Δ - Σ modulator dynamics often limit the generalization capability of classical PINNs, making accurate prediction for high-order designs a challenging task.

Quantum computing offers a different computational method that is particularly suitable for handling complex and high-dimensional problems [7]. Through the principles of superposition and entanglement, quantum systems can represent and process information in exponentially large Hilbert spaces, which offer richer feature representations than classical models [8]. Quantum neural networks (QNNs) have recently emerged as a promising approach for learning nonlinear mappings in high-dimensional spaces. Therefore, QNNs can be naturally extended to mimic complex physical entities, and this leads to the introduction of quantum PINNs (QPINNs) [9]. By exploiting quantum characteristics and combining with current classical machine learning algorithms, QPINNs provide a powerful framework for approximating the behavior of systems governed by complex dynamics. For Δ - Σ modulators, where the relationship between topology, parameters, and performance metrics is highly nonlinear, QPINNs offer a promising solution to capture these interactions more effectively than classical approaches [10].

In this paper, we propose a QPINN framework for efficient prediction of the SNR for high-order Δ - Σ modulators. The proposed model utilizes parameterized quantum circuits to flexibly encode modulator characteristics into quantum states, before being measured and processed by a classical neural network for better control of output prediction. However, the raw topologies will be pre-processed to extract the modulator features before going into the QPINN, to facilitate more efficient learning of the QPINN and cope with the limit in the number of available qubits. In particular, this feature extraction stage reduces the dimensionality of the input space while preserving key dynamical properties of the system, enabling efficient encoding within a limited number of qubits. The results demonstrate that the QPINN achieves high accuracy in SNR prediction while maintaining strong generalization across different topologies. Furthermore, the model exhibits stable training behavior and consistent performance across different data splits, indicating robustness in learning the underlying system dynamics. Quantitative metrics for accuracy of $RMSE$, MAE , R^2 were obtained at 6.937, 5.522, and 0.845, respectively. These results are comparable to strong classical baselines, confirming the effectiveness of the proposed hy-

brid quantum-classical approach. These findings highlight the potential of integrating quantum machine learning techniques into analog circuit design workflows, which can therefore sharply accelerate complicated tasks.

II. Δ - Σ MODULATOR TOPOLOGY AND FEATURE EXTRACTION

A. ABCD Matrix

ABCD matrix consists of four sub-matrices A , B , C , and D , which can generalize a Δ - Σ modulator structure with well-defined relationships among internal signal flows [11]. The modulator is the core component of Δ - Σ ADC, since noise shaping depends entirely on its structure. An example in Fig. 1 is demonstrated to show how a 4th-order modulator topology is transformed into a structured matrix with systematic and coherent columns and rows. Various modulator characteristics can be derived from the ABCD matrix, although this is beyond the scope of this paper. In this work, the ABCD matrix is treated solely as a numerical representation for input into the QPINN.

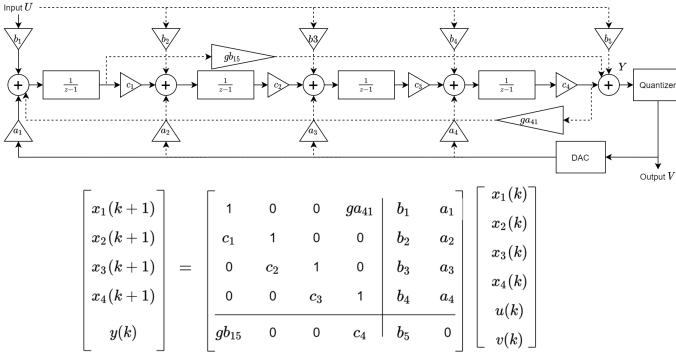


Fig. 1: Example of conversion between Δ - Σ modulator and its corresponding ABCD matrix.

B. Feature Extraction

The ABCD matrix contains a large number of parameters, so directly encoding it into a QPINN is inefficient. Such high dimensionality would require more qubits, so encoding such a large number of parameters into a QPINN is impractical. Therefore, a pre-processing stage is applied to extract a compact set of informative features before feeding data into the QPINN.

Several physically meaningful features can be derived from the ABCD matrix without loss of essential information. These include poles and zeros of the signal transfer function (STF) and noise transfer function (NTF), which directly characterize system stability and noise shaping behavior [12]. In addition, global matrix properties such as trace, determinant, spectral radius, and norms of the submatrices A , B , C , and D provide compact summaries of the system dynamics. Since the ABCD matrix is inherently a structured matrix, it can also be decomposed using singular value decomposition (SVD), allowing the extraction of dominant modes and reducing redundancy in the

representation [13]. These features collectively form a lower-dimensional and physically interpretable feature space that is more suitable for encoding into a QPINN.

After feature extraction, statistical analysis is performed to evaluate the relevance of each feature with respect to the target performance metric, namely the SNR. In particular, the Pearson correlation coefficient (r) is computed between each extracted feature and the SNR over the training dataset [14]. This step identifies the most informative features that exhibit strong relationships with the performance metric. By selecting features with high correlation, the model focuses on the most influential factors governing system behavior, thereby improving training efficiency and generalization. The resulting feature set serves as an optimized input representation for the QPINN framework.

III. QUANTUM PHYSICS-INFORMED NEURAL NETWORKS

The proposed QPINN follows a hybrid quantum-classical neural network architecture designed to efficiently learn the mapping between extracted features and the target performance metric. The model consists of three main components: a classical embedding network, a QNN, and a classical post-processing network. Given an input feature vector, the embedding network first projects the data into a lower-dimensional latent space compatible with the number of qubits. This is implemented using fully connected layers with nonlinear activations, ensuring that the transformed features are properly scaled and bounded before quantum encoding.

Next, the QNN forms the core of the proposed framework (Fig. 2). The encoded features are embedded into quantum states using angle encoding, where each input dimension is mapped to rotation angles (about Y-axis) applied to individual qubits. The quantum circuit is composed of multiple layers of parameterized blocks [15], each consisting of single-qubit rotations followed by entangling operations. Specifically, for each qubit, rotation gates R_X and R_Z are applied with trainable parameters, enabling flexible transformation of the quantum state. These are followed by a chain of controlled-NOT (CNOT) gates that introduce entanglement between neighboring qubits, allowing the circuit to capture correlations across different features. By stacking multiple such layers, the QNN is able to construct a highly expressive nonlinear mapping within the Hilbert space. The final quantum state is measured using expectation values of Pauli- Z operators on each qubit, producing a vector of observables that serves as input to the next stage.

The classical post-processing network takes the quantum measurement outputs and maps them to the final prediction. This stage consists of fully connected layers with nonlinear activations and dropout for regularization, enhancing the model's ability to generalize. The entire architecture is trained end-to-end using gradient-based optimization, where gradients of the quantum circuit are computed via the adjoint differentiation method. By combining classical feature extraction, quantum-enhanced representation learning, and physics-informed inputs, the proposed QPINN provides a compact yet expressive

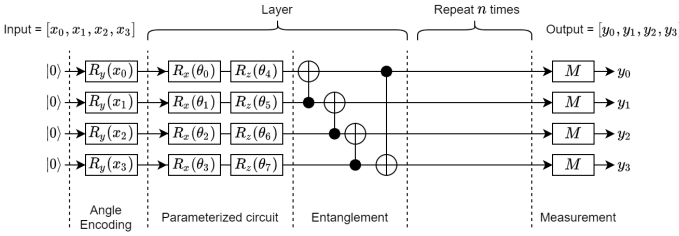


Fig. 2: Quantum circuit representation of the QNN within the proposed network.

model capable of capturing complex relationships in high-dimensional design spaces while maintaining computational efficiency.

IV. SIMULATION RESULTS AND DISCUSSIONS

A. Simulation Settings

The simulation framework is implemented using a hybrid quantum-classical approach. The dataset is first generated through MATLAB simulations, providing labeled samples of Δ - Σ modulator configurations with corresponding SNR values. The quantum circuit is constructed and simulated using PennyLane, which enables efficient differentiation and seamless integration with classical optimization, while the classical components of the model are developed in PyTorch. The dataset is normalized and divided into training and testing sets with an 80/20 split, covering an SNR range from 0 to 100 dB to ensure sufficient variability in the learning process. After the training phase, a calibration step is applied to further refine the trained parameters, improving the model's predictive performance and robustness.

B. Simulation Results

1) *Feature correlation*: The correlation analysis shows that features related to pole dynamics are the most influential for SNR prediction, as shown in Table I. In particular, the largest magnitude of the NTF pole and the spectral radius achieve the highest correlation (0.706), highlighting the importance of pole proximity to the unit circle in noise shaping performance. The trace of A and the mean of NTF poles also exhibit strong correlations (0.658), followed by the determinant of A (0.500). Overall, we used 16 features with the highest correlation with the SNR for feeding into the QPINN.

TABLE I: Extracted features with the highest correlation with SNR

Feature	Correlation value
Largest magnitude of NTF poles	0.706
Spectral radius	0.706
Trace of A	0.658
Mean of NTF poles	0.658
Determinant of A	0.500

2) *Convergence behavior*: The training process exhibits stable and fast convergence, as shown by the decreasing trends of both $RMSE$ and MAE over epochs in Fig. 3. A sharp reduction is observed during the initial training stage, indicating that the model quickly captures the main data patterns. After

approximately 20–30 epochs, both metrics gradually stabilize with only minor fluctuations, suggesting that the model has reached a steady state and avoids overfitting. This behavior confirms the effectiveness of the proposed training framework in achieving reliable and consistent performance.

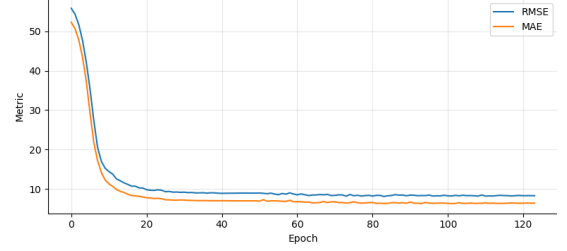


Fig. 3: Validation losses (RMSE/MAE) during training.

3) *Prediction accuracy*: The prediction accuracy of the proposed model is illustrated through the scatter plot of predicted versus true SNR values (Fig. 4). The calibrated predictions (orange points) are more tightly clustered around the ideal diagonal line compared to the raw outputs (blue points), indicating a clear improvement after calibration. This reduction is consistent with the achieved performance metrics of $RMSE = 6.937$ dB and $MAE = 5.522$ dB. Furthermore, the coefficient of determination $R^2 = 0.845$ demonstrates a strong linear agreement between predicted and true SNR values, confirming that the model captures the underlying relationship effectively across a wide SNR range.

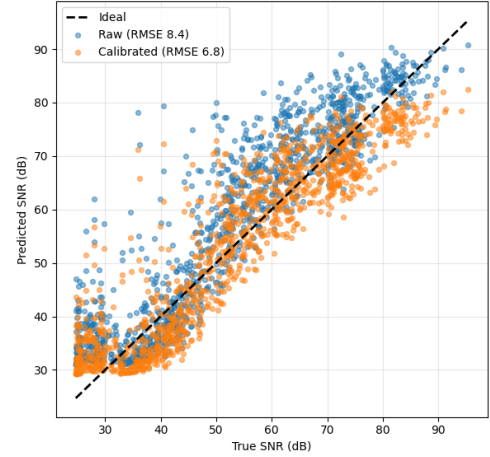


Fig. 4: Scatter plot of predicted versus true SNR values before and after calibration.

Fig. 5 also validates this accuracy by comparing predicted and true SNR values across a smaller range of individual samples. The predictions closely follow the actual line, with only minor deviations at certain points, indicating good generalization and consistency. The model successfully tracks both low and high SNR regions, preserving the overall trend and local variations. This further confirms that the proposed model is capable of capturing both global patterns and fluctuations in the data.

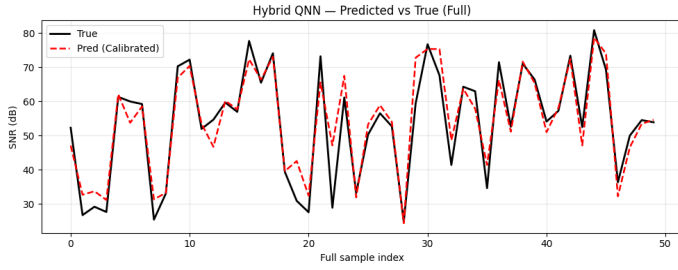


Fig. 5: Sample-wise comparison of predicted and true SNR values.

The results of Table II show the performance of QPINN in comparison with classical baselines of Random Forest (RF) and Deep Neural Network (DNN). The three models exhibit noticeable differences across evaluation metrics. The proposed QPINN achieves the best overall performance, with the lowest $RMSE$ (6.937 dB), lowest MAE (5.522 dB) and highest R^2 (0.845), indicating strong predictive accuracy and robustness to larger errors. In contrast, the DNN baseline shows lower performance ($RMSE = 9.453$ dB, $MAE = 6.977$ dB, $R^2 = 0.768$), suggesting limited effectiveness in capturing the underlying relationships within the feature space. The RF model achieves moderate performance ($RMSE = 8.395$ dB, $MAE = 6.125$ dB, $R^2 = 0.817$), benefiting from nonlinear modeling of engineered features but still underperforming compared to QPINN. This consistent improvement across all metrics highlights the effectiveness of the proposed hybrid quantum-classical framework. It is also noticeable that the R^2 values of all models remain moderate, due to the limited dataset, and predicted values of SNR being in dB scale, where small variations in modulator topology may lead to significant changes in SNR.

TABLE II: Performance comparison of different models on the test set

Model	RMSE (dB)	MAE (dB)	R^2
Random Forest	8.395	6.125	0.817
DNN	9.453	6.977	0.768
QPINN	6.937	5.522	0.845

V. CONCLUSION

The paper presented a quantum physics-informed neural network (QPINN) framework for efficient prediction of high-order Δ - Σ modulator performance. To address the challenges posed by high-dimensional design spaces, we introduced a feature extraction stage that reduces the complexity of the ABCD representation while preserving the most informative system characteristics. By integrating quantum neural networks with physics-based constraints, the proposed model effectively captures the nonlinear relationship between modulator features and SNR. Through careful model design and calibration, the QPINN achieves strong predictive accuracy with reduced computational cost compared to conventional simulation approaches. Extensive experiments demonstrate the effectiveness of the proposed framework, achieving reliable

performance with $RMSE = 6.937$ dB, $MAE = 5.522$ dB, and $R^2 = 0.845$, while maintaining stable convergence during training. The results confirm that quantum-enhanced models can serve as powerful surrogate tools for complex analog circuit evaluation. This work, therefore, highlights the potential of quantum machine learning in addressing high-dimensional optimization problems in analog and mixed-signal systems.

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