

Geometry-Aware Coverage Optimization for Generalized Pinching-Antenna Systems

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Abstract—This paper considers coverage-oriented design for generalized pinching-antenna systems in blockage-rich scenarios. A discrete activation architecture is adopted, in which each waveguide chooses one active pinching point from a finite candidate set. By taking the environment geometry into consideration and introducing a deterministic obstacle-aware line-of-sight indicator, we derive a grid-based average signal-to-noise ratio (SNR) model whose geometry-related coefficients can be obtained offline. On this basis, the design is formulated as an SNR-threshold coverage maximization problem over the service region. We show that this problem admits an exact mixed-integer linear reformulation and is NP-hard. To support large-scale implementation, we further develop a low-complexity waveguide-wise coordinate-update algorithm based on precomputed candidate-to-grid SNR contributions. Simulation results confirm that the proposed geometry-aware activation method consistently outperforms random activation and fixed-array baselines in terms of regional coverage.

Index Terms—Pinching antenna, geometry-aware deployment, coverage maximization

I. INTRODUCTION

In blockage-rich wireless environments, such as warehouses, shopping malls, partitioned halls, and industrial floors, service quality can vary sharply across locations due to geometry-dependent propagation conditions. Conventional fixed-site deployments address this issue mainly through beamforming, power control, and user scheduling, while their effective radiation locations remain unchanged. Consequently, even if the quality of currently served links is improved, geometry-driven coverage gaps may still remain.

Generalized pinching-antenna systems introduce an additional spatial design dimension by allowing the effective radiation point to be adjusted along supporting media, e.g., dielectric waveguides and leaky coaxial cables [1]–[3]. In this way, the network can not only shape beams from predetermined sites, but also alter where signals are effectively emitted or received inside the service region. This capability is particularly attractive for network-level design in irregular environments, where spatially flexible radiation can better exploit favorable LoS corridors and mitigate blockage-induced dead zones.

Recent studies have shown that generalized pinching antennas can improve spectral efficiency [4], [5], energy efficiency

[6], [7], and outage performance [8], [9]. Correspondingly, blockage-aware pinching-antenna system designs have attracted growing attention [10]–[19]. Existing works mainly follow two directions. One direction uses probabilistic blockage models to characterize LoS availability and NLoS fading, and investigates their impact on performance as well as blockage-aware placement and beamforming for given user sets [10], [12], [13]. The other direction considers blockage-aware designs under energy-harvesting objectives [15] and the study of hardware effects including in-waveguide attenuation [17]. Beyond these statistical models, recent efforts have moved toward geometry-/map-aware blockage modeling for obstacle-rich environments. For example, a deterministic obstacle-based model for geometry-aware pinching-antenna placement was developed in [18]. Recently, an environment-division multiple access scheme by leveraging the ability of pinching antennas to reconfigure LoS links was proposed in [19]. Nevertheless, these works are still link-level, in the sense that the system is optimized for given user sets and instantaneous user geometry. Such formulations are sensitive to localization errors, and may require frequent updates when users move.

Motivated by the above discussion, this paper considers geometry-aware coverage design for dielectric-waveguide-based generalized pinching-antenna systems in blockage-rich environments. We employ a discrete activation model, where each waveguide contains a finite number of candidate pinching-antenna locations and exactly one of them is selected. By gridizing the service region and explicitly modeling the environment, we derive a tractable geometry-aware per-grid average-SNR expression using a deterministic LoS visibility test, with all geometry-related terms precomputable offline. On this basis, we formulate an average-SNR-threshold coverage maximization problem, obtain an equivalent mixed-integer linear programming form, and establish its NP-hardness. We further propose a structure-aware coordinate-ascent method for efficient optimization. Simulation results verify that the proposed activation design achieves clear coverage gains over random-activation and fixed-array benchmarks under a variety of blockage layouts and parameter settings.

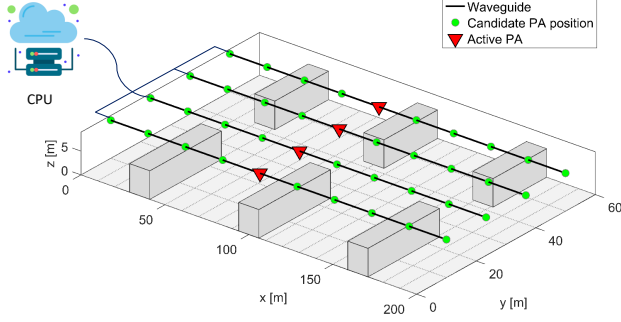


Fig. 1: Illustration of the considered geometry-aware pinching-antenna system with blockages and candidate activation points.

II. SYSTEM DESCRIPTION AND GEOMETRY-DEPENDENT SNR MODEL

This section introduces the considered dielectric-waveguide-based generalized pinching-antenna architecture, the blockage-aware channel model, and the resulting grid-level average-SNR expression used in the subsequent coverage optimization.

A. Discrete Activation Architecture

Consider a service region of size $D_x \times D_y$. As shown in Fig. 1, N dielectric waveguides are mounted below the ceiling at height d_v , parallel to the x -axis and uniformly spaced along the y -direction. Let $\mathcal{N} = \{1, \dots, N\}$ denote the waveguide index set. The distance between adjacent waveguides is $d_h = D_y/(N-1)$, and the feed point of waveguide $n \in \mathcal{N}$ is $\psi_{0,n} = [0, (n-1)d_h - D_y/2, d_v]^\top$.

We adopt a discrete activation architecture, where each waveguide has M candidate pinching-antenna locations. Let $\mathcal{M} = \{1, \dots, M\}$ denote the candidate index set. Candidate $m \in \mathcal{M}$ on waveguide n is located at $\psi_{n,m}^{\text{pin}} = [\tilde{x}_{n,m}, (n-1)d_h - D_y/2, d_v]^\top$, where $\tilde{x}_{n,m} \in [0, D_x]$. Binary variables $a_{n,m} \in \{0, 1\}$ are introduced to indicate activation, where $a_{n,m} = 1$ means that candidate (n, m) is selected. We assume one candidate is activated on each waveguide for transmission, and thus

$$\sum_{m=1}^M a_{n,m} = 1, \quad \forall n \in \mathcal{N}. \quad (1)$$

Collecting all binary variables gives the activation matrix $\mathbf{A} \triangleq [a_{n,m}]_{n \in \mathcal{N}, m \in \mathcal{M}}$, which is the design variable of this paper.

For network-level evaluation, we discretize the service region into an $N_h \times N_v$ grid. The grid spacings are $\Delta_x = D_x/N_h$ and $\Delta_y = D_y/N_v$. Accordingly, the center of grid (u, v) is

$$x_u = \left(u - \frac{1}{2}\right) \Delta_x, \quad y_v = -\frac{D_y}{2} + \left(v - \frac{1}{2}\right) \Delta_y, \quad (2)$$

for $u = 1, \dots, N_h$ and $v = 1, \dots, N_v$. We use $\psi_{u,v} = [x_u, y_v, 0]^\top$ to denote the representative location of grid (u, v) , and define the grid index set as

$$\Omega = \{1, \dots, N_h\} \times \{1, \dots, N_v\}. \quad (3)$$

This discrete candidate model is convenient for geometry-aware optimization, since once the blockage layout is fixed, the channel contributions from all candidates to all grids can be precomputed offline. The online design then only needs to determine the binary activation matrix $\mathbf{A}$.

B. Blockage-Aware Channel Model

We model the environment by K rectangular cuboid obstacles. Let $\mathcal{K} \triangleq \{1, \dots, K\}$ denote the obstacle index set. Obstacle $k \in \mathcal{K}$ occupies the volume

$$\mathcal{B}_k = [x_k^{\min}, x_k^{\max}] \times [y_k^{\min}, y_k^{\max}] \times [0, H_{\text{blk}}], \quad (4)$$

where $H_{\text{blk}} < d_v$ is the obstacle height.

For candidate (n, m) and grid (u, v) , let $h_{n,m}(\psi_{u,v})$ denote the corresponding complex channel coefficient. We adopt a geometry-aware LoS/NLoS model of the form

$$h_{n,m}(\psi_{u,v}) = \chi_{n,m}(\psi_{u,v}) h_{n,m}^{\text{LoS}}(\psi_{u,v}) + h_{n,m}^{\text{NLoS}}(\psi_{u,v}), \quad (5)$$

where $\chi_{n,m}(\psi_{u,v}) \in \{0, 1\}$ is a deterministic visibility indicator. More precisely, $\chi_{n,m}(\psi_{u,v}) = 1$ if the line segment between $\psi_{n,m}^{\text{pin}}$ and $\psi_{u,v}$ does not intersect any obstacle, and $\chi_{n,m}(\psi_{u,v}) = 0$ otherwise.

Following [20], the LoS channel between candidate (n, m) and grid (u, v) is expressed as

$$h_{n,m}^{\text{LoS}}(\psi_{u,v}) = \frac{\sqrt{\eta} \exp\left(-j \frac{2\pi}{\lambda} r_{n,m}(\psi_{u,v}) + j \frac{2\pi}{\lambda_g} \tilde{x}_{n,m}\right)}{r_{n,m}(\psi_{u,v})}, \quad (6)$$

where λ and λ_g denote the free-space and guided wavelengths, respectively, and $\eta = c^2/(4\pi f_c)^2$ with c being the speed of light and f_c the carrier frequency. The link distance is

$$r_{n,m}(\psi_{u,v}) = \sqrt{(x_u - \tilde{x}_{n,m})^2 + C_{n,v}}, \quad (7)$$

with

$$C_{n,v} = \left(y_v - ((n-1)d_h - D_y/2)\right)^2 + d_v^2. \quad (8)$$

The NLoS component accounts for the combined scattered paths and is modeled as

$$h_{n,m}^{\text{NLoS}}(\psi_{u,v}) = \sum_{\ell=1}^{N_c} g_{n,m,\ell}(\psi_{u,v}), \quad (9)$$

where N_c is the number of scattering clusters and

$$g_{n,m,\ell}(\psi_{u,v}) \sim \mathcal{CN}\left(0, \frac{\mu_\ell^2}{r_{n,m}^2(\psi_{u,v})}\right). \quad (10)$$

The total NLoS power coefficient is then given by $\mu^2 = \sum_{\ell=1}^{N_c} \mu_\ell^2$.

C. Grid-Level Average SNR

For a given activation matrix $\mathbf{A}$, the channel contributed by waveguide n to grid (u, v) is

$$\tilde{h}_n(\psi_{u,v}; \mathbf{A}) \triangleq \sum_{m=1}^M a_{n,m} h_{n,m}(\psi_{u,v}), \quad (11)$$

and the corresponding channel vector is $\tilde{\mathbf{h}}(\psi_{u,v}; \mathbf{A}) = [\tilde{h}_1(\psi_{u,v}; \mathbf{A}), \dots, \tilde{h}_N(\psi_{u,v}; \mathbf{A})]^\top$.

Let s be the transmitted symbol with $\mathbb{E}[|s|^2] = 1$. Then the received signal at grid (u, v) can be written as

$$y(\psi_{u,v}) = \sqrt{P} \tilde{\mathbf{h}}^\top(\psi_{u,v}; \mathbf{A}) \mathbf{w}(\psi_{u,v}; \mathbf{A}) s + z_{u,v}, \quad (12)$$

where P denotes the transmit power and $z_{u,v} \sim \mathcal{CN}(0, \sigma^2)$ is the additive noise. We use maximum-ratio transmission, i.e.,

$$\mathbf{w}(\psi_{u,v}; \mathbf{A}) = \frac{\tilde{\mathbf{h}}^*(\psi_{u,v}; \mathbf{A})}{\|\tilde{\mathbf{h}}(\psi_{u,v}; \mathbf{A})\|}. \quad (13)$$

Accordingly, the received SNR at grid (u, v) is

$$\Gamma(\psi_{u,v}; \mathbf{A}) = \rho \|\tilde{\mathbf{h}}(\psi_{u,v}; \mathbf{A})\|^2, \quad (14)$$

where $\rho = P/\sigma^2$. Since the NLoS component is random, we use the average received SNR as the grid-level performance metric.

Lemma 1. *For any activation matrix $\mathbf{A}$, the average SNR at grid $(u, v) \in \Omega$ is*

$$\bar{\Gamma}(\psi_{u,v}; \mathbf{A}) = \rho \sum_{n=1}^N \sum_{m=1}^M a_{n,m} \frac{\chi_{n,m}(\psi_{u,v})\eta + \mu^2}{r_{n,m}^2(\psi_{u,v})}. \quad (15)$$

Proof: The result follows by taking expectation over the NLoS fading in (14), using the zero mean and independence of the NLoS components, together with the one-hot activation structure in (1) [21]. ■

Lemma 1 shows that the grid-level average SNR depends linearly on the activation variables. Moreover, the coefficients $\frac{\chi_{n,m}(\psi_{u,v})\eta + \mu^2}{r_{n,m}^2(\psi_{u,v})}$ depend only on the environment geometry and the candidate/grid locations. Hence, these terms can be precomputed once the obstacle layout is known. In the next section, we use (15) to formulate a geometry-aware coverage maximization problem.

III. COVERAGE MAXIMIZATION UNDER AN SNR THRESHOLD

Based on the grid-level average-SNR model in (15), we next optimize the activation matrix so as to maximize the covered portion of the service region.

A. Coverage Definition and Optimization Problem

For a prescribed SNR threshold $\gamma_{\text{th}} > 0$, we say that grid $(u, v) \in \Omega$ is covered under activation matrix $\mathbf{A}$ if

$$\bar{\Gamma}(\psi_{u,v}; \mathbf{A}) \geq \gamma_{\text{th}}. \quad (16)$$

Define the corresponding binary indicator as

$$c_{u,v}(\mathbf{A}) \triangleq \mathbb{1}\{\bar{\Gamma}(\psi_{u,v}; \mathbf{A}) \geq \gamma_{\text{th}}\}, \quad (u, v) \in \Omega. \quad (17)$$

The total number of covered grids is then $\sum_{(u,v) \in \Omega} c_{u,v}(\mathbf{A})$.

Accordingly, the geometry-aware coverage optimization problem is formulated as

$$\max_{\mathbf{A}} \sum_{(u,v) \in \Omega} c_{u,v}(\mathbf{A}) \quad (18a)$$

$$\text{s.t.} \quad \sum_{m=1}^M a_{n,m} = 1, \quad \forall n \in \mathcal{N}, \quad (18b)$$

$$a_{n,m} \in \{0, 1\}, \quad \forall n \in \mathcal{N}, m \in \mathcal{M}. \quad (18c)$$

This problem aims to select one candidate on each waveguide so that the number of grids whose average SNR exceeds γ_{th} is maximized.

B. Equivalent Mixed-Integer Linear Programming (MILP) Reformulation

The difficulty of problem (18) lies in the discontinuous indicator function in the objective. To handle this, we introduce auxiliary binary variables $\{c_{u,v}\}_{(u,v) \in \Omega}$ and enforce the coverage condition by

$$\bar{\Gamma}(\psi_{u,v}; \mathbf{A}) \geq \gamma_{\text{th}} c_{u,v}, \quad \forall (u, v) \in \Omega. \quad (19)$$

Because $\bar{\Gamma}(\psi_{u,v}; \mathbf{A}) \geq 0$, the above relation does not require any big- M parameter. Then, using (15), problem (18) can be rewritten as

$$\max_{\mathbf{A}, \{c_{u,v}\}} \sum_{(u,v) \in \Omega} c_{u,v} \quad (20a)$$

$$\text{s.t.} \quad \bar{\Gamma}(\psi_{u,v}; \mathbf{A}) \geq \gamma_{\text{th}} c_{u,v}, \quad \forall (u, v) \in \Omega, \quad (20b)$$

$$\sum_{m=1}^M a_{n,m} = 1, \quad \forall n \in \mathcal{N}, \quad (20c)$$

$$a_{n,m} \in \{0, 1\}, \quad \forall n \in \mathcal{N}, m \in \mathcal{M}, \quad (20d)$$

$$c_{u,v} \in \{0, 1\}, \quad \forall (u, v) \in \Omega. \quad (20e)$$

This is an exact MILP representation of the original coverage maximization problem. However, the problem is still combinatorial in nature.

Proposition 1. *Problem (20) is NP-hard.*

Proof: See Appendix A. ■

While (20) can be handled by standard MILP solvers for moderate sizes, its complexity grows rapidly with the number of grids and candidate locations. We next develop an alternative algorithm to solve problem (20) in a low-complexity manner.

C. Waveguide-Wise Coordinate-Ascent Method

The proposed algorithm updates the activation of one waveguide at a time while keeping the others fixed. Its efficiency relies on the additive structure of the average-SNR expression in (15).

Suppose that candidate m_n is currently active on waveguide n , i.e., $a_{n,m_n} = 1$. Define the residual average SNR excluding waveguide n as

$$\bar{\Gamma}_{-n}(\psi_{u,v}; \mathbf{A}) \triangleq \bar{\Gamma}(\psi_{u,v}; \mathbf{A}) - \rho \bar{G}_{n,m_n}(\psi_{u,v}), \quad (u, v) \in \Omega, \quad (21)$$

where

$$\bar{G}_{n,m}(\psi_{u,v}) \triangleq \frac{\chi_{n,m}(\psi_{u,v})\eta + \mu^2}{r_{n,m}^2(\psi_{u,v})}. \quad (22)$$

If waveguide n switches from m_n to candidate m , then the updated SNR at grid (u, v) becomes

$$\bar{\Gamma}(\psi_{u,v}; \mathbf{A}_{n \rightarrow m}) = \bar{\Gamma}_{-n}(\psi_{u,v}; \mathbf{A}) + \rho \bar{G}_{n,m}(\psi_{u,v}), \quad (23)$$

where $\mathbf{A}_{n \rightarrow m}$ denotes the activation matrix after replacing the n th row by the choice of candidate m .

For waveguide n , we therefore select the candidate that maximizes the covered-grid count:

$$m_n^* \in \arg \max_{m \in \mathcal{M}} \sum_{(u,v) \in \Omega} \mathbb{1}\{\bar{\Gamma}_{-n}(\psi_{u,v}; \mathbf{A}) + \rho \bar{G}_{n,m}(\psi_{u,v}) \geq \gamma_{\text{th}}\}. \quad (24)$$

When several candidates produce the same covered-grid count, we use the total post-threshold margin

$$\sum_{(u,v) \in \Omega} \left[\bar{\Gamma}_{-n}(\psi_{u,v}; \mathbf{A}) + \rho \bar{G}_{n,m}(\psi_{u,v}) - \gamma_{\text{th}} \right]^+ \quad (25)$$

as a tie-breaker, where $[x]^+ \triangleq \max\{x, 0\}$.

Starting from any feasible initialization, the algorithm updates each waveguide iteratively according to (24). Since each step optimizes over the finite candidate set of one waveguide while holding the others fixed, the covered-grid count never decreases. Because the feasible set is finite, the procedure terminates after a finite number of updates.

Each waveguide update evaluates M candidates over $|\Omega|$ grids, so one sweep has complexity $\mathcal{O}(NM|\Omega|)$. Since all candidate-to-grid SNR contributions are precomputed offline, the online operations consist only of additions, comparisons, and candidate selection.

IV. SIMULATION RESULTS

Numerical experiments are provided to assess the proposed geometry-aware coverage design for generalized pinching-antenna systems. Unless otherwise stated, the simulation parameters follow Table I. The propagation environment includes $K = 6$ rectangular cuboid obstacles, each with height $H_{\text{blk}} < d_v$. A candidate pinching-antenna location and a grid

TABLE I: Default simulation parameters.

Parameter	Value
Carrier frequency (f_c)	28 GHz
Communication-region size ($D_x \times D_y$)	$200 \times 60 \text{ m}^2$
Waveguide height (d_v)	10 m
Transmit power (P)	40 dBm
Noise power (σ^2)	-70 dBm
NLoS power (μ^2)	-60 dBm
Number of PAs / waveguides (N)	4
Number of grids ($N_h \times N_v$)	400×120
Obstacle height (H_{blk})	6 m

point are regarded as LoS-blocked whenever the straight three-dimensional segment connecting them intersects any obstacle. The blockage configuration is fixed as

$$\mathcal{O}(x_0, y_a, y_b) \triangleq [x_0 - \delta_x, x_0 + \delta_x] \times [y_a, y_b] \times [0, H_{\text{blk}}], \quad (26)$$

where $x_0 \in \{12, 38, 64\}$, $\delta_x = 4 \text{ m}$, and $(y_a, y_b) \in \{(0, 18), (33, 48)\}$.

A. Coverage Probability versus SNR Threshold

We first examine the average-SNR-threshold coverage probability, defined as the fraction of valid grids whose average received SNR is no smaller than γ_{th} . Three schemes are considered: the proposed coordinate-ascent-based activation, a random-activation benchmark with one uniformly selected candidate per waveguide, and a fixed-array benchmark with an N -element half-wavelength-spaced array placed at the region center. Fig. 2 plots the coverage probability versus γ_{th} for $(N, M) = (4, 10)$. The proposed design performs best over most of the threshold range, showing that geometry-aware candidate selection effectively improves regional coverage. Random activation yields intermediate performance, indicating that the pinching-antenna architecture remains beneficial even without optimization. The fixed-array benchmark performs worst at low and moderate thresholds because it lacks spatial adaptability in blockage-rich environments. At very high thresholds, however, it can slightly outperform the pinching-antenna schemes by forming a localized high-SNR region near the center. By contrast, the proposed design spreads radiation points over multiple waveguides and locations, which is better for broad coverage but may produce a slightly smaller area above an extremely high threshold.

Fig. 3 shows the case of a larger system with $(N, M) = (8, 20)$. Relative to Fig. 2, the coverage curves are lifted across the whole threshold range, with the most visible improvement appearing at low and moderate thresholds. This trend is natural, since increasing the number of waveguides enhances power accumulation and spatial diversity, while a larger candidate set provides finer spatial control over the activated radiation points. Consequently, the optimized scheme can more effectively align with favorable propagation paths and alleviate blockage-induced weak-SNR regions. By contrast, in

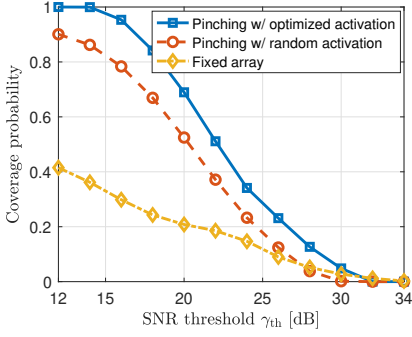


Fig. 2: Coverage probability versus the SNR threshold γ_{th} for $(N, M) = (4, 10)$.

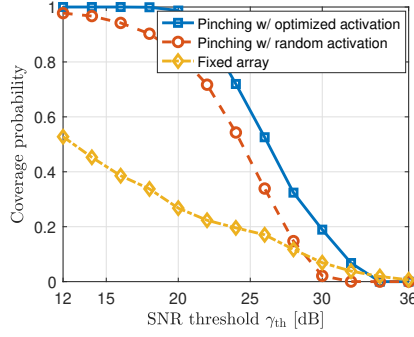


Fig. 3: Coverage probability versus the SNR threshold γ_{th} for $(N, M) = (8, 20)$.

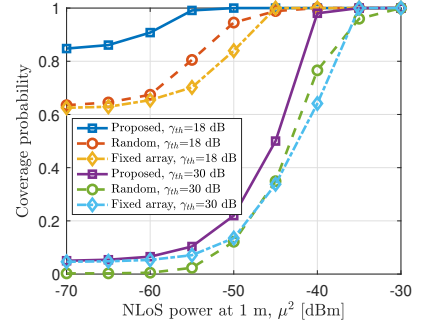


Fig. 4: Coverage probability versus the NLoS power parameter μ^2 under two representative SNR thresholds.

the very high-threshold regime, the improvement becomes less pronounced because the coverage is mainly determined by a small number of localized strong-SNR spots. These results indicate that geometry-aware activation is especially beneficial for practical region-wide coverage enhancement under moderate SNR targets.

B. Impact of the NLoS Power Level

Fig. 4 reports how the coverage probability changes with the aggregate NLoS power level μ^2 under two target thresholds, namely $\gamma_{th} = 18$ dB and $\gamma_{th} = 30$ dB. For all three schemes, increasing μ^2 improves the coverage, because stronger scattered components raise the average received SNR at locations where the direct path is weak or blocked. When the target is moderate, i.e., $\gamma_{th} = 18$ dB, the proposed geometry-aware design already achieves a high coverage level over a broad range of μ^2 , which suggests that it can rely more on favorable spatial activation than on a strong NLoS contribution. In contrast, for the more demanding threshold $\gamma_{th} = 30$ dB, the coverage becomes much more dependent on μ^2 , since meeting such a target typically requires either strong visible links or sufficiently strong scattered power. As μ^2 grows further, all schemes approach near-full coverage and their differences gradually narrow.

V. CONCLUSION

We studied regional coverage enhancement for generalized pinching-antenna systems under explicit blockage geometry. Using a discrete candidate-selection architecture, we established a tractable grid-based average-SNR model in which the environment-dependent coefficients are obtained offline from the obstacle layout. This led to an SNR-threshold coverage maximization problem over the activation matrix. For this problem, we presented an exact MILP representation, proved NP-hardness, and developed a lightweight waveguide-wise coordinate update method for efficient optimization. Numerical results showed clear and consistent coverage gains relative to random activation and a conventional fixed-array deployment across different SNR targets, candidate resolutions, and NLoS strengths. Overall, the results demonstrate that blockage-aware activation of pinching antennas provides an effective mecha-

nism for improving area-wide coverage in complex wireless environments.

APPENDIX A PROOF OF PROPOSITION 1

We prove the NP-hardness of problem (20) by a polynomial-time reduction from the classical NP-hard *Maximum Coverage* (MC) problem.

A. MC Problem

Consider a ground set $\mathcal{U} = \{1, 2, \dots, G\}$, and a family of subsets $\mathcal{S}_1, \mathcal{S}_2, \dots, \mathcal{S}_J, \mathcal{S}_j \subseteq \mathcal{U}$. Given an integer budget k , the MC problem asks for the selection of at most k subsets whose union contains the largest possible number of elements of $\mathcal{U}$ [22]. Equivalently, we may use the exact- k form

$$\max_{\mathcal{J} \subseteq \{1, \dots, J\}, |\mathcal{J}|=k} \left| \bigcup_{j \in \mathcal{J}} \mathcal{S}_j \right|. \quad (27)$$

The exact- k form is sufficient here because, if fewer than k subsets are selected, one can always add arbitrary extra subsets without reducing the union size.

B. Construction of a Coverage-Optimization Instance

Given an arbitrary instance of (27), we construct a special instance of problem (20) according to the following steps.

1) *Grid set, waveguide and candidates*: Create G grids, one for each element of $\mathcal{U}$. Thus, the grid index set Ω satisfies $|\Omega| = G$. For convenience, we identify grid $i \in \Omega$ with element $i \in \mathcal{U}$. Set the number of waveguides to $N = k$. On each waveguide, define $M = J$ candidate activation positions. Candidate index $m \in \{1, \dots, M\}$ corresponds to subset $\mathcal{S}_m$.

2) *Precomputed SNR gains*: Fix the SNR threshold γ_{th} used in problem (20). For each grid $i \in \Omega$, waveguide $n \in \mathcal{N}$, and candidate $m \in \mathcal{M}$, define the precomputed gain

$$\bar{G}_{n,m}(i) \triangleq \begin{cases} \gamma_{th}/\rho, & \text{if } i \in \mathcal{S}_m, \\ 0, & \text{if } i \notin \mathcal{S}_m, \end{cases} \quad (28)$$

where $\rho = P/\sigma^2$ is the transmit SNR. Under this construction, if candidate m is activated on any waveguide, then grid i receives exactly γ_{th} average-SNR contribution whenever $i \in \mathcal{S}_m$, and zero otherwise.

3) *Resulting average SNR*: For any feasible activation matrix $\mathbf{A} = [a_{n,m}]$, the average SNR at grid i is

$$\bar{\Gamma}(i; \mathbf{A}) = \rho \sum_{n=1}^N \sum_{m=1}^M a_{n,m} \bar{G}_{n,m}(i). \quad (29)$$

By the one-hot constraint, each waveguide activates exactly one candidate, so the total number of activated candidate indices is exactly $N = k$, possibly with repetitions.

C. Key Equivalence

Let $\mathcal{J}(\mathbf{A})$ denote the set of distinct candidate indices selected by $\mathbf{A}$, i.e.,

$$\mathcal{J}(\mathbf{A}) \triangleq \{m \in \{1, \dots, M\} | a_{n,m} = 1 \text{ for at least one } n\}. \quad (30)$$

Clearly, we have $|\mathcal{J}(\mathbf{A})| \leq k$.

Now consider any grid $i \in \Omega$. Using (28) and (29), we have $\bar{\Gamma}(i; \mathbf{A}) \geq \gamma_{\text{th}}$ if and only if there exists at least one activated candidate index $m \in \mathcal{J}(\mathbf{A})$ such that $i \in \mathcal{S}_m$. Therefore, grid i is covered exactly when the corresponding element of $\mathcal{U}$ belongs to the union of the selected subsets. Hence, the covered-grid set induced by $\mathbf{A}$ is precisely

$$\bigcup_{m \in \mathcal{J}(\mathbf{A})} \mathcal{S}_m. \quad (31)$$

As a result, the objective value of problem (20) for this constructed instance is exactly the cardinality of the union of the subsets indexed by $\mathcal{J}(\mathbf{A})$.

D. Two Directions of the Reduction

We next show that the constructed coverage problem and the MC instance are equivalent in objective value.

A) From MC to the constructed coverage problem: Take any feasible solution $\mathcal{J} \subseteq \{1, \dots, J\}$ of (27) with $|\mathcal{J}| = k$. Since $N = k$, we can assign one selected subset index from $\mathcal{J}$ to each waveguide and activate the corresponding candidate. This yields a feasible activation matrix $\mathbf{A}$. By construction, the covered grids are exactly $\bigcup_{j \in \mathcal{J}} \mathcal{S}_j$, so the number of covered grids equals the objective value of the MC solution.

B) From the constructed coverage problem to MC: Conversely, take any feasible activation matrix $\mathbf{A}$ for the constructed instance. The distinct activated candidate indices form the set $\mathcal{J}(\mathbf{A})$, which satisfies $|\mathcal{J}(\mathbf{A})| \leq k$. If $|\mathcal{J}(\mathbf{A})| < k$, we can add arbitrary extra subset indices to obtain a set $\hat{\mathcal{J}}(\mathbf{A})$ with exactly k elements. Since adding subsets cannot decrease the union size, we have

$$\left| \bigcup_{m \in \hat{\mathcal{J}}(\mathbf{A})} \mathcal{S}_m \right| \geq \left| \bigcup_{m \in \mathcal{J}(\mathbf{A})} \mathcal{S}_m \right|. \quad (32)$$

Using (31), the right-hand side is exactly the number of covered grids under $\mathbf{A}$. Therefore, from any feasible activation matrix, we can construct a feasible MC solution with objective value no smaller than the corresponding coverage objective. In summary, the above construction is polynomial in G , J , and k , and preserves the optimal objective value. Hence, a polynomial-time solver for problem (20) would imply a polynomial-time solver for the MC problem. Since MC is NP-hard, problem (20) is also NP-hard. This completes the proof. ■

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