Joint Optimal Beamforming and Discrete Phase Shift Design in STAR-RIS with Quantum Optimization

(Invited Paper)

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***Abstract*—In this paper, we investigate the potential of a near-optimal hybrid quantum-classical optimization approach to jointly optimizing beamforming and discrete phase shifts of the simultaneous transmitting and reflecting reconfigurable intelligent surface (STAR-RIS) assisted wireless network. We first formulate a discrete optimization problem to maximize the total power transmitted to the ground users by optimizing the beamforming at the base station (BS) and STAR-RIS phase shift cells under minimal power allocation for each user and the power budget at the BS. Then, we propose a quantum approximate optimization algorithm with alternating optimization (QAOA-AO) method that iteratively addresses beamforming components and discrete phase shifts to search for the near-optimal solutions for the problem. Numerical results validate the effectiveness and robustness of the proposed QAOA-AO compared to the classical benchmarks, and further highlight its potential for practical applicability for solving medium-to-large-scale optimization problems.**

I. Introduction

The sixth generation (6G) mobile network is expected to be revolutionary in new wireless applications, including the integration of physical things, human activities, and digital technologies into a cyber-physical ecosystem where the physical and digital worlds are globally converged [1]. 6G will also act as a flexible platform for future services by improving traditional aspects such as capacity, coverage, speed, and latency, as well as new indicators such as availability, reliability, predictability, network resilience, and trustworthiness. Unfortunately, these advancements cannot be fully achieved by enabling fifth generation (5G) techniques such as massive multiple-input multiple-output (MIMO), millimeter wave (mmWave) communication or ultra-dense networks (UDN), even though they have significantly enhanced the performance of the above criteria to reach 5G key performance indicators (KPIs). As a result, deep research on innovative technologies is needed to inherit the use cases already covered by 5G and overcome the limitations of the 5G timeframe.

Simultaneous transmitting and reflecting reconfigurable intelligent surface (STAR-RIS) has recently emerged as a potential 6G development technology to further enhance the spectrum and energy efficiency [1]. In short, a STAR-RIS is a two-dimensional (2D) programmable metasurface composed of many small unit cells such as meta atoms or simply STAR-RIS elements. These low-cost elements can individually adjust their phase shifts, amplitudes, or polarizations of the incoming electromagnetic waves with low power to reconfigure wireless propagation channels [2]. By this unique mechanics, the STAR-RIS technology can be applied to assist wireless communications and mobile networks by integrating with other methodologies, such as non-orthogonal multiple access (NOMA), orthogonal multiple access (OMA) [3], and ISAC [4]. In particular, [3], the authors addressed a narrow-band STAR-RIS aided downlink communication network with multiple access schemes such as NOMA and OMA. Regarding the ISAC technique, the authors in [4] proposed a system model with a dual-functional BS (DFBS) that operates in full-duplex mode to simultaneously provide communication services to multiple users connected with a single antenna and perform target sensing.

Quantum approximate optimization algorithm (QAOA) has recently arisen as a high performance tool to address NP-hard mixed-integer problems (MIPs). In short, QAOA is a hybrid quantum-classical approach in which the angle parameters of its quantum circuit layers are iteratively optimized using a classical computer, while the solution is derived from the measurement outcomes of the QAOA circuit executed on a quantum computer. Broad studies on QAOA applications have verified its validity and robustness in different fields such as vehicle routing [5], electric vehicle charging [6], and edge computing [7]. These findings show clear proof that quantum optimization can be useful in many areas, including STAR-RIS technology and the next-generation mobile network overall.

Motivated by these aforementioned findings, in this paper, we introduce a near-optimal hybrid QAOA-AO method to address a joint beamforming and discrete phase shift optimal design in a STAR-RIS-assisted wireless communications network. We first formulate a comprehensive problem for maximizing the sum received signal power by jointly optimizing the BS power allocation and the STAR-RIS discrete phase shifts. Then, the hybrid QAOA-AO method is used to iteratively tackle the beamforming subproblem by the plain QAOA and the phase shifts subproblem by deriving a closed-form expression.

II. System Model and Problem Formulation

A. System Model

In this study, we consider a STAR-RIS aided downlink (DL) communication system, as shown in Fig. 1. Herein, a terrestrial BS with M antennas is used to transmit information to N terrestrial single-antenna users with the help of a STAR-RIS implemented as an active beamforming relay to transmit signals to users. The number of antennas in the scattering elements of the STAR-RIS is set as P. The set of these on-ground users, the BS antennas, and the reflecting elements can now be denoted as $\mathcal{N} \triangleq \{1, \dots, N\}$, $\mathcal{M} \triangleq \{1, \dots, M\}$, and $\mathcal{P} \triangleq \{1, \dots, P\}$, respectively.

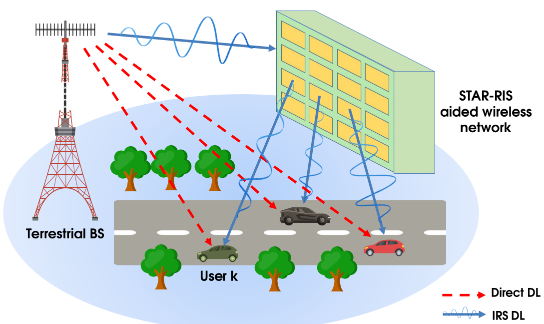

Fig. 1: An illustration of a STAR-RIS-assisted wireless network designed to serve users.

Let us denote $h^H_{i,pm} \in \mathbb{C}$ as the channel coefficient of the antenna $m \in \mathcal{M}$ to the STAR-RIS element $p \in \mathcal{P}$, $\mathbf{h^H_{i,p}} \triangleq \{h^H_{i,pm}\}_{m \in \mathcal{M}} \in \mathbf{C}^{1\times M}$ and $\mathbf{H^H_I} \triangleq \{\mathbf{h^H_{i,p}}\} \in \mathbb{C}^{P\times M}$ as the channel gain vector coming out of element p and matrix of channel coefficients from the BS to the STAR-RIS, respectively. Similarly, $h^H_{r,np} \in \mathbb{C}$ is defined as the channel gain from the reflecting element $p \in \mathcal{P}$ to user $n \in \mathcal{N}$, and $\mathbf{h^H_{r,n}} \triangleq \{h^H_{r,np}\} \in \mathbb{C}^{1\times P}$ denotes the channel gain vector from the STAR-RIS to user n. The channel gain of the direct DL from antenna $m \in \mathcal{M}$ to user $n \in \mathcal{N}$ can be assigned as $h^H_{d,nm}$, and $\mathbf{h^H_{d,n}} \triangleq \{h^H_{d,nm}\} \in \mathbb{C}^{1\times M}$. For simplicity, the channel coefficient between components p and q are assumed to follow the independent small-scale fading and can be written as $h^H_{X,pq} = d^{-\varphi_X/2}_{X,pq}\tilde{h}$, where φ_X denotes the path loss exponent in different channels, h is the Rayleigh small-scale fading, i.e., $\tilde{h} \sim \mathcal{CN}(0,1)$. The letter $X \in \{i, r, d\}$ is used to distinguish channel categories and $d_{X,pq}$ is the Euclidean distance between components p and q.

Let $\boldsymbol{\theta} \triangleq [\theta_1, \dots, \theta_P]$ be the set of phase shifts of the scattering elements in STAR-RIS, and for simplicity, we set the amplitudes of these elements equal to 1. Furthermore, the phase shift θ_p of the $p-$th STAR-RIS cell is also restricted to $\{0, \pi\}$. Let $\boldsymbol{\Theta} = \text{diag}\left\{e^{j\theta_1}, \dots, e^{j\theta_P}\right\}$ be the parameters that characterize the way STAR-RIS elements modify the original signal wave.

The complex transmitted signal in the BS can be expressed as $\mathbf{x} = \sum_{n\in\mathcal{N}} \mathbf{w}_n s_n$, where s_n defined as the data needed to transfer to user n and $\mathbf{w}_n \triangleq \{\mathrm{w}_{nm}\} \in \mathbb{C}^{M\times 1}$ is the associated beamforming vector. It is also presumed that the signals transferred $s_n, \forall n \in \mathcal{N}$ are independent random distributed variables with zero mean and unit variance. Now, the signal transmitted from the terrestrial BS to user n can be written as

$$\begin{aligned} y_n &= y^d_n + y^r_n \\ &= \mathbf{h^H_{d,n}}\mathbf{x} + \mathbf{h^H_{r,n}}\boldsymbol{\Theta}\mathbf{H^H_I}\mathbf{x} + z_n \\ &= \left(\mathbf{h^H_{d,n}} + \mathbf{h^H_{r,n}}\boldsymbol{\Theta}\mathbf{H^H_I}\right)\sum_{n\in\mathcal{N}} \mathbf{w_n} s_n + z_n, \end{aligned} \quad (1)$$

where z_n is the additive white Gaussian noise (AWGN) with zero mean and variance σ^2. Following (1), the total achievable signal power of users can be constructed as

$$P_T(\mathbf{w}, \boldsymbol{\theta}) = \sum_{n\in\mathcal{N}} \left|\left(\mathbf{h^H_{d,n}} + \mathbf{h^H_{r,n}}\boldsymbol{\Theta}\mathbf{H^H_I}\right)\mathbf{w_n}\right|^2, \quad (2)$$

where $\mathbf{w} \triangleq \{\mathbf{w_n}\}_{n\in\mathcal{N}}$.

B. Optimization Problem Formulation

In this research, we focus on maximizing the total received signal power by jointly optimizing the power allocation of the BS $\mathbf{w}$ and the STAR-RIS phase shifts $\boldsymbol{\theta}$. The total superimposed signal problem can be formulated as follows:

$$\max_{\mathbf{w},\boldsymbol{\theta}} \ P_T(\mathbf{w}, \boldsymbol{\theta}), \quad (3a)$$

$$\text{s.t.} \ \|\mathbf{w}\|^2 \le P_{\max}, \quad (3b)$$

$$\|\mathbf{w_n}\|^2 \ge P_{\min}, \forall n \in \mathcal{N}. \quad (3c)$$

$$\theta_p \in \{0, \pi\}, \forall p \in \mathcal{P}. \quad (3d)$$

In the above problem, the objective (3a) maximize the total signal power of the ground users. The first constraint (3b) limits the maximum power transferred from the BS, and the second constraint (3c) ensures that the BS allocates the transmission power to each user and the minimum power must not be lower than $P_{\min}$. Finally, constraint (3d) represents discrete variables θ_p.

III. The proposed QAOA-AO solution

Note that the optimized value of $P_T(\mathbf{w}, \boldsymbol{\theta})$ can only be reached when the equality of constraint (3b) is satisfied, i.e., $\|\mathbf{w}\|^2 = P_{\max}$ because of the scalability [8]. Therefore, the primary problem can be reconstructed as follows:

$$\max_{\mathbf{w},\boldsymbol{\theta}} \ P_T(\mathbf{w}, \boldsymbol{\theta}), \tag{4a}$$

$$\text{s.t. } \|\mathbf{w}\|^2 = P_{\max}, \tag{4b}$$

$$\|\mathbf{w_n}\|^2 \geq P_{\min}, \forall n \in \mathcal{N}. \tag{4c}$$

$$\theta_p \in \{0, \pi\}, \forall p \in \mathcal{P}. \tag{4d}$$

Although we have reduced the primary problem, this problem is still difficult to solve due to the discrete variables $\boldsymbol{\theta}$. Moreover, the problem is NP-hard, i.e., extremely challenging and time-consuming to find the optimal value computationally. Hence, we first apply the AO method to split the problem into two subproblems. The subproblem with the beamforming vector being fixed will be converted into a Hamiltonian expression, which can be efficiently solved by the QAOA approach and a quantum processor. For the subproblem with the STAR-RIS elements being fixed, we obtain the optimal beamforming power directly due to the quadratic form of the objective.

A. Beamforming Optimization Subproblem

This subproblem is created by optimizing the BS beamforming factor $\mathbf{w}$ while $\boldsymbol{\theta} = \boldsymbol{\theta}^{(t)}$ remains constant. Let us first denote $\boldsymbol{\Theta}^{(t)} \triangleq \text{diag}\left\{e^{j\theta_1^{(t)}}, \ldots, e^{j\theta_P^{(t)}}\right\}$ and $P_{T,\mathbf{w}}^{(t)}(\mathbf{w}) \triangleq \sum_{n\in\mathcal{N}} \left|\left(\mathbf{h_{d,n}^H} + \mathbf{h_{r,n}^H}\boldsymbol{\Theta}^{(t)}\mathbf{H_I^H}\right)\mathbf{w}_n\right|^2$. The problem can now be demonstrated as

$$\max_{\mathbf{w}} \ P_{T,\mathbf{w}}^{(t)}(\mathbf{w}), \tag{5a}$$

$$\text{s.t. } \|\mathbf{w}\|^2 = P_{\max}. \tag{5b}$$

$$\|\mathbf{w_n}\|^2 \geq P_{\min}, \forall n \in \mathcal{N}. \tag{5c}$$

In order to simplify the problem given above, let us denote $\left(\mathbf{v_n^H}\right)^{(t)} \triangleq \mathbf{h_{d,n}^H} + \mathbf{h_{r,n}^H}\boldsymbol{\Theta}^{(t)}\mathbf{H_I^H}$ and $(\mathbf{v_n})^{(t)} \triangleq (\mathbf{h_{d,n}^H} + \mathbf{h_{r,n}^H}\boldsymbol{\Theta}^{(t)}\mathbf{H_I^H})^{\mathbf{H}} \in \mathbb{C}^{M\times 1}$. We also rewrite $\mathbf{w_n} = \alpha_n\mathbf{p_n}$, where α_n is the beamforming amplitude and $\mathbf{p_n}$ denotes the beamforming direction of $\mathbf{w_n}$. These beamforming factors must follow the constraint (5b), (5c) and can be equivalently rewritten as

$$\sum_{n\in\mathcal{N}} \alpha_n^2 = P_{\max}, \tag{6a}$$

$$\alpha_n^2 \geq P_{\min}, \tag{6b}$$

$$\|\mathbf{p_n}\| = 1. \tag{6c}$$

Taking into account these denotations, the objective $P_{T,\mathbf{w}}^{(t)}(\mathbf{w})$ can be rewritten as

$$P_{T,\mathbf{w}}^{(t)} = \sum_{n\in\mathcal{N}} |\left(\mathbf{v_n^H}\right)^{(t)}\mathbf{w_n}|^2 = \sum_{n\in\mathcal{N}} \alpha_n^2\mathbf{p_n^H}\mathbf{V_n}^{(t)}\mathbf{p_n}, \tag{7}$$

where $\mathbf{V_n}^{(t)} \triangleq (\mathbf{v_n})^{(t)}\left(\mathbf{v_n^H}\right)^{(t)} \in \mathbb{C}^{M\times M}$. For constant α_n, the maximum value of (7) is $P_{\max}\lambda_{\max}^{n,(t+1)}$ and can be achieved at $\mathbf{p_n} = \mathbf{q}_{\max}^{n,(t+1)}$, where $\lambda_{\max}^{n,(t+1)}$ is the largest eigenvalue of the matrix $\mathbf{V_n}^{(t)}$ and $\mathbf{q}_{\max}^{n,(t+1)}$ is the corresponding eigenvector of $\mathbf{V_n}^{(t)}$.

The other factor we need to optimize is the beamforming amplitude α_n. From (6a) and (6b), the beamforming amplitude can always be bounded as

$$\sqrt{P_{\min}} \leq \alpha_n \leq \sqrt{P_{\max} - (N-1)P_{\min}}. \tag{8}$$

We now denote $j = \arg\max_{n\in\mathcal{N}}\{\lambda_{\max}^{n,(t+1)}\}$, which means that $\lambda_{\max}^{j,(t+1)} \geq \lambda_{\max}^{n,(t+1)}, \forall n \in \mathcal{N}$ and $j \in \mathcal{N}$. Hence, we have the following inequality demonstrated as

$$\begin{aligned} P_{T,\mathbf{w}}^{(t)} &\leq \sum_{n\in\mathcal{N}} \alpha_n^2\lambda_{\max}^{n,(t+1)} \\ &\leq (P_{\max} - (N-1)P_{\min})\lambda_{\max}^{j,(t+1)} + \sum_{\substack{n\in\mathcal{N}\\ n\neq j}} P_{min}\lambda_{\max}^{n,(t+1)}. \end{aligned} \tag{9}$$

The inequality holds if and only if $\alpha_n = \sqrt{P_{\min}}, \forall n \neq j$ and $\alpha_j = \sqrt{P_{\max} - (N-1)P_{\min}}$. Hence, the maximizer $\mathbf{w}^{(t+1)}$ of the subproblem (5) can now be formulated as

$$\mathbf{w_n}^{(t+1)} = \alpha_n\mathbf{q}_{\max}^{n,(t+1)}, \forall n \in \mathcal{N}. \tag{10}$$

B. Phase Shift Optimization Subproblem

To formulate this subproblem, let us first set $\mathbf{w} = \mathbf{w^{(t)}} \triangleq \{\mathbf{w}_n^{(t)}\}$ as a feasible solution of (4). We also denote $P_{T,\boldsymbol{\theta}}^{(t)}(\boldsymbol{\theta}) \triangleq \sum_{n\in\mathcal{N}} \left|\left(\mathbf{h_{d,n}^H} + \mathbf{h_{r,n}^H}\boldsymbol{\Theta}\mathbf{H_I^H}\right)\mathbf{w}_n^{(t)}\right|^2$. From this denotation, we reformulate the subproblem as

$$\max_{\boldsymbol{\theta}} \ P_{T,\boldsymbol{\theta}}^{(t)}(\boldsymbol{\theta}), \tag{11a}$$

$$\text{s.t. } \theta_p \in \{0, \pi\}, \forall p \in \mathcal{P}. \tag{11b}$$

This subproblem is a discrete optimization problem with the discrete variable $\boldsymbol{\theta}$. To achieve the Hamiltonian formulation, we first need to reformulate this subproblem into a QUBO expression. Generally, a QUBO problem can be described as

$$\min_{\mathbf{x}} \ \mathbf{x}^{\mathbf{T}}\mathbf{Q}\mathbf{x}, \tag{12a}$$

$$\text{s.t. } x_i \in \{0,1\}, \forall x_i \in \mathbf{x}. \tag{12b}$$

In this general form, $\mathbf{x} \in \{0,1\}^n$ is the binary vector and $\mathbf{Q} \in \mathbb{R}^{n\times n}$ is a symmetric matrix that describes the linear terms of the variables in $\mathbf{x}$ and the interactions between two different decision variables in $\mathbf{x}$. Since our subproblem is the maximization problem, we first convert it into a minimization problem as

$$\min_{\boldsymbol{\theta}} \ -P_{T,\boldsymbol{\theta}}^{(t)}(\boldsymbol{\theta}), \tag{13a}$$

$$\text{s.t. } \theta_p \in \{0,\pi\}, \forall p \in \mathcal{P}. \tag{13b}$$

To simplify the objective in (13a), $P_{T,\boldsymbol{\theta}}^{(t)}(\boldsymbol{\theta})$ can be separated as the sum of the signal power components as follows:

$$-P_{T,\boldsymbol{\theta}}^{(t)}(\boldsymbol{\theta}) = -\sum_{n\in\mathcal{N}} Q_{n,\boldsymbol{\theta}}^{(t)}, \tag{14}$$

where

$$Q_{n,\boldsymbol{\theta}}^{(t)} \triangleq \left| \sum_{m\in\mathcal{M}} \left(h_{d,nm}^H + \sum_{p\in\mathcal{P}} h_{r,np}^H e^{j\theta_p} h_{i,pm}^H \right) \mathrm{w}_{nm}^{(t)} \right|^2. \tag{15}$$

It is worth noting that $e^{j\theta_p} = \cos\theta_p$ since $\sin\theta_p$ is always equal to 0. Following this, by defining $T_{nm}^{r,(t)} \triangleq \Re\{h_{d,nm}^H \mathrm{w}_{nm}^{(t)}\}, T_{nm}^{i,(t)} \triangleq \Im\{h_{d,nm}^H \mathrm{w}_{nm}^{(t)}\}$, $S_{p,nm}^{r,(t)} \triangleq \Re\{h_{r,np}^H h_{i,pm}^H \mathrm{w}_{nm}^{(t)}\}, S_{p,nm}^{i,(t)} \triangleq \Im\{h_{r,np}^H h_{i,pm}^H \mathrm{w}_{nm}^{(t)}\}$, $\forall n \in \mathcal{N}$, and $\mathcal{T}_n^r \triangleq \sum_{m\in\mathcal{M}} T_{nm}^{r,(t)}, \mathcal{T}_n^{i,(t)} \triangleq \sum_{m\in\mathcal{M}} T_{nm}^{i,(t)}$, $\mathcal{S}_{p,n}^{r,(t)} \triangleq \sum_{m\in\mathcal{M}} S_{p,nm}^{r,(t)}, \mathcal{S}_{p,n}^{i,(t)} \triangleq \sum_{m\in\mathcal{M}} S_{p,nm}^{i,(t)}$, $Q_{n,\boldsymbol{\theta}}^{(t)}$ can be rewritten as

$$\begin{aligned} Q_{n,\boldsymbol{\theta}}^{(t)} &= (\mathcal{T}_n^{r,(t)})^2 + (\mathcal{T}_n^{i,(t)})^2 \\ &+ \sum_{p\in\mathcal{P}} \left[(\mathcal{S}_{p,n}^{r,(t)})^2 + (\mathcal{S}_{p,n}^{i,(t)})^2 \right] \\ &+ 2\sum_{p\in\mathcal{P}} \cos\theta_p (\mathcal{S}_{p,n}^{i,(t)} \mathcal{T}_n^{i,(t)} + \mathcal{S}_{p,n}^{r,(t)} \mathcal{T}_n^{r,(t)}) \\ &+ \sum_{\substack{p,q\in\mathcal{P}\\p\neq q}} \cos\theta_p \cos\theta_q (\mathcal{S}_{p,n}^{r,(t)} \mathcal{S}_{q,n}^{r,(t)} + \mathcal{S}_{p,n}^{i,(t)} \mathcal{S}_{q,n}^{i,(t)}). \end{aligned} \tag{16}$$

This formulation can be attained due to the fact that $\cos^2\theta_p = 1, \forall\theta_p \in \{0,\pi\}$. Since $\cos\theta_p \in \{1,-1\} \cos\theta_p \in \{1,-1\}, \forall\theta_p \in \{0,\pi\}$, we can introduce binary variables $a_p \in \{0,1\}$ to express θ_p as $\theta_p = a_p\pi, \forall p \in \mathcal{P}$. We can also represent the term $\cos\theta_p$ by using a_p as

$$\cos(a_p\pi) = 1 - 2a_p. \tag{17}$$

Using (17), (16) can be reformulated as

$$\begin{aligned} Q_{n,\boldsymbol{\theta}}^{(t)} &= \sum_{p\in\mathcal{P}} a_p \left(-4\mathcal{A}_{p,n}^{(t)} - 4\sum_{q\neq p} \mathcal{B}_{pq,n}^{(t)} \right) \\ &+ 4\sum_{\substack{p,q\in\mathcal{P}\\p\neq q}} a_p a_q \mathcal{B}_{pq,n}^{(t)} + \mathcal{C}_n^{(t)}, \end{aligned} \tag{18}$$

where $\mathcal{A}_{p,n}^{(t)} \triangleq \mathcal{S}_{p,n}^{i,(t)} \mathcal{T}_n^{i,(t)} + \mathcal{S}_{p,n}^{r,(t)} \mathcal{T}_n^{r,(t)}$, $\mathcal{B}_{pq,n}^{(t)} \triangleq \mathcal{S}_{p,n}^{r,(t)} \mathcal{S}_{q,n}^{r,(t)} + \mathcal{S}_{p,n}^{i,(t)} \mathcal{S}_{q,n}^{i,(t)}$, and $\mathcal{C}_n^{(t)} \triangleq (\mathcal{T}_n^{r,(t)})^2 + (\mathcal{T}_n^{i,(t)})^2 + \sum_{p\in\mathcal{P}} \left[(\mathcal{S}_{p,n}^{r,(t)})^2 + (\mathcal{S}_{p,n}^{i,(t)})^2 \right] + 2\sum_{p\in\mathcal{P}} \mathcal{A}_{p,n}^{(t)} + \sum_{\substack{p,q\in\mathcal{P}\\p\neq q}} \mathcal{B}_{pq,n}^{(t)}$. From the developments above, our initial objective $-P_{T,\boldsymbol{\theta}}^{(t)}(\boldsymbol{\theta})$ can be rewritten as

$$\begin{aligned} -P_{T,\boldsymbol{\theta}}^{(t)} &= \sum_{p\in\mathcal{P}} a_p \sum_{n\in\mathcal{N}} \left(4\mathcal{A}_{p,n}^{(t)} + 4\sum_{q\neq p} \mathcal{B}_{pq,n}^{(t)} \right) \\ &- 4\sum_{\substack{p,q\in\mathcal{P}\\p\neq q}} a_p a_q \sum_{n\in\mathcal{N}} \mathcal{B}_{pq,n}^{(t)} - \sum_{n\in\mathcal{N}} \mathcal{C}_n^{(t)}. \end{aligned} \tag{19}$$

The formulation in (19) has now been converted to a QUBO structure with binary vector $\mathbf{a} \triangleq \{a_p\}_{p\in\mathcal{P}}$ and our coefficient matrix $\mathbf{Q}$ can be expressed as

$$\begin{aligned} Q_{pp} &= \sum_{n\in\mathcal{N}} \left(4\mathcal{A}_{p,n}^{(t)} + 4\sum_{q\neq p} \mathcal{B}_{pq,n}^{(t)} \right), \forall p \in \mathcal{P}, \\ Q_{pq} &= -4\sum_{n\in\mathcal{N}} \mathcal{B}_{pq,n}^{(t)}, \forall p,q \in \mathcal{P}, p \neq q. \end{aligned} \tag{20}$$

Herein, Q_{pq} is the component of $\mathbf{Q}$ at row p, column q. The following step of the process is to convert the QUBO problem in (19) into the Hamiltonian expression. To handle this work, we introduce spin variables $z_p = 1-2a_p, \forall p \in \mathcal{P}$, and convert the binary decision variable a_p to z_p by the equation given as

$$a_p = \frac{1-z_p}{2}, \forall p \in \mathcal{P}. \tag{21}$$

Based on the preceding developments above, we implement our Hamiltonian by substituting (21) into (19) and terminate the constant term as

$$H_{\text{state}} \triangleq -2\sum_{p\in\mathcal{P}} z_p \sum_{n\in\mathcal{N}} \mathcal{A}_{p,n}^{(t)} - \sum_{\substack{p,q\in\mathcal{P}\\p\neq q}} z_p z_q \sum_{n\in\mathcal{N}} \mathcal{B}_{pq,n}^{(t)}. \tag{22}$$

Following the above developments, we propose a QAOA-based optimization algorithm to tackle this problem, as illustrated in Algorithm 1. The algorithm starts by initializing some parameters that are required for the algorithm, i.e. the known parameters in the mathematical model, feasible $\mathbf{w}^{(\mathbf{t})}$ for the system, and appropriate quantum backend specifications. Subsequently, a parameterized quantum circuit (ansatz) is designed due to the known Hamiltonian objective and M layers for measurements. For each layer m, we set an initial parameter vector (β_m, γ_m) and set $(\boldsymbol{\beta}, \boldsymbol{\gamma}) \triangleq (\beta_m, \gamma_m)_{m=1,2,\ldots,M}$. Next, a classical optimizer is employed to minimize the objective value with respect to the variational parameters $(\boldsymbol{\beta}, \boldsymbol{\gamma})$. The algorithm then optimizes and updates the parameters iteratively until convergence or reaches the maximum number of iterations.

Algorithm 1 : Proposed quantum-centric optimization approach for solving (11).

1: **Input**: $M, N, P, \mathbf{H_I^H}, \mathbf{h_{r,n}^H}, \mathbf{h_{d,n}^H}, \varphi_i, \varphi_r, \varphi_d, \tilde{h}, \sigma^2$, locations of components in wireless network; $\mathbf{w^{(t)}}$, quantum backend settings; classical optimizer.
2: Build a parameterized ansatz circuit with M layers and an initial vector $(\boldsymbol{\beta}, \boldsymbol{\gamma})$.
3: Exploit a classical optimizer for optimizing and updating parameters iteratively.
4: Insert the parameterized vector $(\boldsymbol{\beta}, \boldsymbol{\gamma})$ into the ansatz circuit.
5: **repeat**
6: Optimize and update $(\boldsymbol{\beta}, \boldsymbol{\gamma})$ by the optimizer chosen in Step 3
7: Measure the expectation value of the quantum state $\langle \boldsymbol{\beta}, \boldsymbol{\gamma} | H_{\text{state}} | \boldsymbol{\beta}, \boldsymbol{\gamma} \rangle$ and seek for convergence.
8: **until** convergence
9: **Output:** Optimized STAR-RIS phase shifts $\boldsymbol{\theta}^{(t+1)}$ constructed by binary variables and the optimal value of the objective $P_{T,\boldsymbol{\theta}}^{(t+1)}(\boldsymbol{\theta}^{(t+1)})$.

C. Proposed QAOA-AO Algorithm

To formulate the QAOA-AO algorithm, let us denote $P_T^{(t)} \triangleq P_T(\mathbf{w}^{(t)}, \boldsymbol{\theta}^{(t)})$ as the maximum value of (3) at iteration t. As shown in Algorithm 2, we initialize feasible solutions of (3) as $\boldsymbol{\theta}^{(0)}, \mathbf{w}^{(0)}$, setting $t = 0$ and some appropriate parameters given in Algorithm 1. Then, at iteration t, the optimal beamforming $\mathbf{w}^{(t+1)}$ can be directly attained from Section III-A. Next, from Algorithm 1, we derive the new optimized phase shifts $\boldsymbol{\theta}^{(t+1)}$ and update $\boldsymbol{\Theta}^{(t+1)}$ from Section III-B. Eventually, the objective can be updated to $P_T^{(t+1)}$, the counting index is then updated to $t = t + 1$ and the QAOA-AO algorithm continues to operate until the convergence condition is reached.

IV. Simulation Results and Discussions

A. Simulation Settings

In this section, we present the simulation results of our proposed algorithm using the following specific parameters. The number of antennas implemented in the BS is $M = 8$, and the number of terrestrial users is

Algorithm 2 : Proposed QAOA-AO Algorithm for solving (3).

1: **Initialization**: Set $t = 0$, maximum number of iterations, $N_{\max}$; generate the initial feasible points $\boldsymbol{\theta}^{(0)}$ with the corresponding optimized $\mathbf{w}^{(0)}$, and choose the known parameters in Algorithm 1.
2: **while** $(P_T^{(t)} > P_T^{(t-1)}$or $t \leq N_{\max})$ **do**
3: Use (10) to find the optimal beamforming vectors of the BS $\mathbf{w}^{(t+1)}$ from $\boldsymbol{\theta}^{(t)}$;
4: Use Algorithm 1 to obtain the optimal STAR-RIS phase shifts variables $\boldsymbol{\theta}^{(t+1)}$ from $\mathbf{w}^{(t+1)}$;
5: Calculate $P_T^{(t+1)}$ based on the optimal variables $\boldsymbol{\theta}^{(t+1)}$ and $\mathbf{w}^{(t+1)}$;
6: Update $t = t + 1$;
7: **end while**
8: **Output**: near-optimal solutions of $\boldsymbol{\theta}^*$ and $\mathbf{w}^*$ and the near-maximum total achievable rate P_T^*.

$N = 3$. The location of terrestrial BS attached to M antennas is presumably located at $[0, 0, 50]$ meters, and terrestrial users have their position fixed at $[100, 50, 0]$, $[180, -40, 0]$ and $[-75, 120, 0]$ meters, respectively. The location of the STAR-RIS center is assumed to be $[75, 50, 20]$ meters. The maximum power emitted by the BS is set to 40 dBm, and the minimum power allocated to each user is set to 27 dBm. In addition, the path loss exponents of different channel types are set to $\varphi_i = 2.2, \varphi_r = 2.6, \varphi_d = 3.5$, and the noise power is $\sigma^2 = -90$ dBm. Afterwards, the number of qubits used in the optimization problem is in the range of $[3, 18]$, and the number of random Rayleigh channels simulated in this work is 50.

B. Numerical Results and Discussions

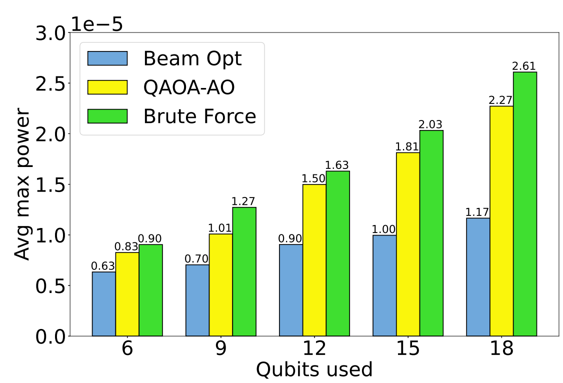

Fig. 2: Enhancement of total power transmitted from BS to users with the help of STAR-RIS.

Fig. 2 shows the improvement in total transmission power to ground users in STAR-RIS aided wireless networks. Theoretically, the QAOA-AO algorithm ensures that the solution converges at a local optimum point; therefore, the performance of the QAOA-AO algorithm might be lower than the exhaustive search method since they always derive the global optimum point. However, the numerical results of these two methodologies are quite close. In fact, there are many cases where the results of the QAOA-AO and brute force methods are equivalent to each other in both theory and simulation. Moreover, the performance of the QAOA-AO method is superior to that of the only optimizing beamforming method, which validates the efficiency of this proposed method.

Fig 3 illustrates the exponential growth of the running time ratio between the QAOA-AO and the brute force methodologies, considering the same increase of variables from 6 to 18 qubits. The time ratio is taken by the time running of 6 to 18 qubits scenario divided by that of the 3 qubit case. As we can see, the time ratio from the brute force approach is enormous when compared with the time ratio of the QAOA-AO method, especially in the higher qubit cases. The increasing speed of the brute force is also higher than that of the QAOA-AO algorithm, which indicates that when the number of qubits increases exponentially, the brute force method will take much more time than the QAOA-AO. This result also validates the robustness of the proposed QAOA-AO method and further validates the potential that the operation time of the hybrid quantum-classical methods can be much smaller than the classical computing, especially with problems that are NP-hard or intractable.

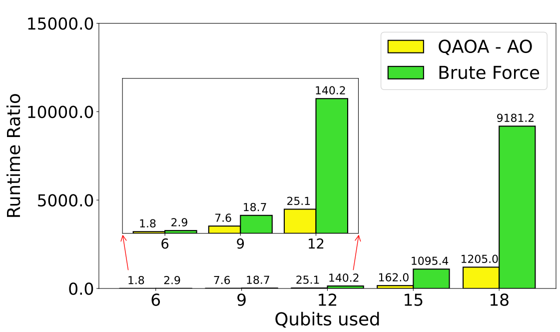

Fig. 3: The time ratio of QAOA-AO algorithm compared with brute force for qubits in range $[6, 18]$.

V. Conclusion

In this paper, we have investigated the integrated challenge of addressing the STAR-RIS optimization problem using the proposed QAOA-AO methodology, in order to provide a clear perspective on our proposed solutions. Numerical solutions demonstrate that our proposed QAOA-AO method generally surpasses classical exhaustive search in run time efficiency for sophisticated problems which seem classically intractable. The QAOA-AO algorithm also achieves near-optimal solutions compared to the brute force method and outperforms the beam optimization method. These findings underscore the strong potential of hybrid quantum-classical approaches in handling discrete phase shift STAR-RIS problems, which is more practical than those of continuous phase shift. Looking ahead, future work could explore more advanced quantum-classical hybrid techniques—such as adaptive variational algorithms to enhance both the performance and scalability of quantum optimization for STAR-RIS phase design.

Acknowledgments

The work of T. Q. Duong was supported in part by the Canada Excellence Research Chair (CERC) Program CERC-2022-00109, in part by the Natural Sciences and Engineering Research Council of Canada (NSERC) Discovery Grant Program RGPIN-2025-04941, and in part by the NSERC CREATE program (Grant number 596205-2025). The work of B. Canberk is supported in part by The Scientific and Technological Research Council of Turkey (TUBITAK) Frontier R&D Laboratories Support Program for BTS Advanced AI Hub: BTS Autonomous Networks and Data Innovation Lab Project 5239903.

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